Physically-Based Dynamic Morphing of Beam Sounds
(a power-balanced formulation)

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Project Hamecmopsys (https://hamecmopsys.ens2m.fr/)

Motivation

1. Theoretical issues

- find **damping models** that preserve eigen modes
- design **nonlinear** dampings in such a class
- provide a **power balanced formulation** that is preserved in **simulations**

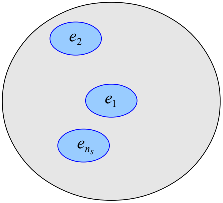
2. Application in musical acoustics

Build **physical models** to produce:

- a **variety of beam sounds** (glokenspiel, xylophone, marimba, etc)
- **morphed sounds** through some extrapolations based on **physical grounds**
  (e.g. meta-materials with damping depending on the magnitude)

Outline

(Ingredient 1) A physical system is made of ...



- Energy-storing components: *(energy)*
  $E = \sum_{n=1}^{N} e_n \geq 0$

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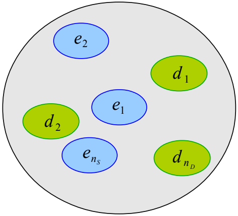

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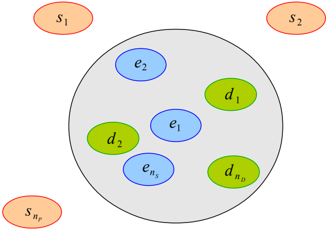

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- External sources: *(external power)*
  $P_{\mathrm{ext}} = \sum_{p=1}^{P} s_p$

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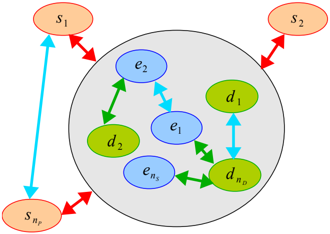

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- Conservative connections *(power balance)*
  $\frac{dE}{dt} = -Q + P_{\mathrm{ext}}$

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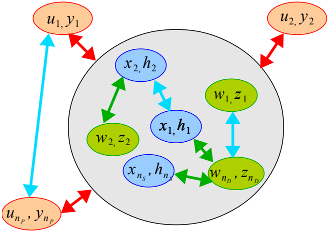

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  (effort × flux : force × velocity, voltage × current, etc)
- External sources: *(external power)*
  $P_{\mathrm{ext}} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^{P} u_p y_p$
- Conservative connections *(power balance)*
  $0 = \nabla H(x)^T \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} + \mathbf{z}(\mathbf{w})^{\mathsf{T}} \cdot \mathbf{w} - \mathbf{u}^T \cdot \mathbf{y}$

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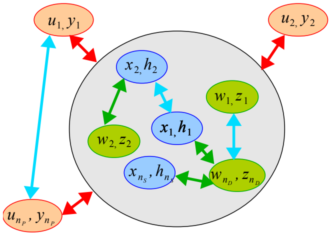

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Port-Hamiltonian Formulation **Power balance**

$$\begin{pmatrix} \frac{d\mathbf{x}}{dt} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix} = S. \begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}$$

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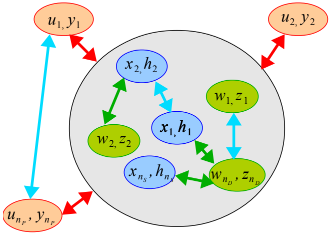

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Port-Hamiltonian Formulation

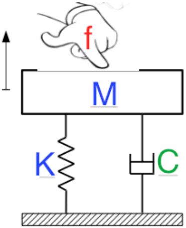
$$\underbrace{\begin{pmatrix} \frac{d\mathbf{x}}{dt} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix}}_{B} = S . \underbrace{\begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}}_{A}$$

Power balance

$$\boxed{\begin{aligned} 0 &= A^T B \\ &= A^T S A \end{aligned}}$$

if $S = -S^T$

Example: damped mechanical oscillator



$x = \begin{bmatrix} M\dot{q} \\ q \end{bmatrix}$	*momentum* *position*	$H(x) = \frac{1}{2} x^T \begin{bmatrix} M^{-1} & 0 \\ 0 & K \end{bmatrix} x$	*kinetic* *potential*
$\frac{\mathrm{d}x}{\mathrm{d}t} = \begin{bmatrix} M\ddot{q} \\ \dot{q} \end{bmatrix}$	*inertia force* *velocity*	$\nabla H(x) = \begin{bmatrix} \dot{q} \\ Kq \end{bmatrix}$	*velocity* *spring force*
$w = \dot{q}$	*velocity*	$z(w) = C\dot{q}$	*damping force*
$y = \dot{q}$	*velocity*	$u = f$	*external force*

$$M\ddot{q} + C\dot{q} + Kq = f$$

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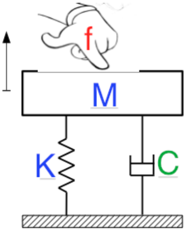

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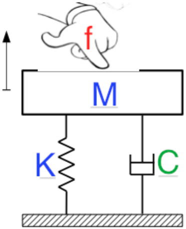

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N coupled oscillators ? q: vector

Matrices: $M = M^T > 0$, $K = K^T \geq 0$, $C = C^T \geq 0$, $1 \equiv I_N$

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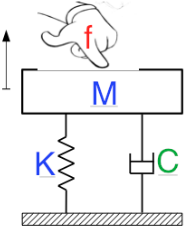

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Passive-guaranteed simulations ? The power balance is preserved, if

$$\frac{\mathrm{d}x}{\mathrm{d}t} \to \frac{x(k+1) - x(k)}{T} \quad \text{and} \quad [\nabla H(x)]_n \to \frac{H_n\big(x_n(k+1)\big) - H_n\big(x_n(k)\big)}{x(k+1) - x(k)}$$

including for nonlinear systems **[Lopes et al., IFAC-LHMNLC'2015]**

(Ingredient 2) Damping models for $M\ddot{q} + C\dot{q} + Kq = f$

Conservative problem (C=0)

- $\ddot{q} + (M^{-1}K)q = M^{-1}f$
- Eigen-modes e_i: $(M^{-1}K)e_i = \omega_i^2 e_i$ (ω_i: pulsation)

Damping that preserves eigen-modes ?

- Choose $M^{-1}C$ as a non-negative polynomial of matrix $M^{-1}K$

$\rightarrow$ *Caughey class (1960)*: $C = c_0 M + c_1 K + c_2 K M^{-1} K + \dots$

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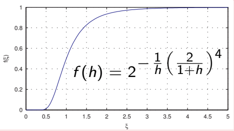
Eigen-modes with nonlinear dynamics ?

- Make c_n depend on the state

Ex.: damping as a function of energy $c_n(x) = \kappa_n\big(H(x)\big) \in [c_n^-, c_n^+]$

- Increasing: $\kappa_n(h) = c_n^- + (c_n^+ - c_n^-) f\left(\frac{h}{h_0}\right)$
- Decreasing: $\kappa_l(h) = c_n^+ - (c_n^+ - c_n^-) f\left(\frac{h}{h_0}\right)$



Application: the Euler-Bernoulli beam

1. Pinned beam excited by a distributed force

(H1) Euler-Bernoulli kinematics: straight cross-section after deformation

(H2) linear approximation for the conservative problem

(H3) viscous and structural dampings: only $c_0, c_1 \geq 0$

2. Dimensionless model (q: deflection, $t \geq 0$, $0 \leq \ell \leq 1$)

- **PDE:** $\underbrace{\partial_t^2 q}_{\mathcal{M} \equiv Id} + \underbrace{(c_0 + c_1 \partial_\ell^4)}_{\mathcal{C}} \partial_t q + \underbrace{\partial_\ell^4}_{\mathcal{K}} q = f_{\mathrm{ext}}$
- **Boundaries** $\ell \in \{0, 1\}$**:** fixed extremities ($q = 0$), no momentum ($\partial_\ell^2 q = 0$)
- **Energy:** $E = \displaystyle\int_0^1 \left(\frac{(\partial_\ell^2 q)^2}{2} + \frac{(\partial_t q)^2}{2} \right) \mathrm{d}\ell$

3. Modal decomposition: $e_m(\ell) = \sqrt{2}\sin(k_m \ell)$ ($k_m = m\pi$, $1 \leq m \leq n$)

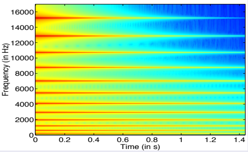
PHS with $x = [q; \dot{q}]$, $q = [q_1, \ldots, q_n]^T$, $u = [u_1, \ldots, u_n]^T$ *(projection of f_{ext})*

where $M = I_n$, $K = \pi^4 \mathrm{diag}(1, \ldots, n)^4$, and $C = c_0 I_n + c_1 K$

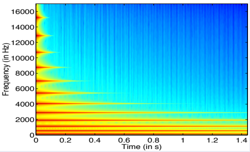
Damping and simulation parameters

Examples of spectrograms for standard linear dampings: $c_0 \sim 10^{-2}$

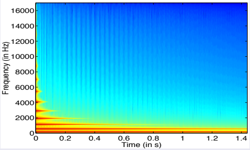
metal ($c_1 \sim 10^{-6}$)



glass ($c_1 \sim 10^{-5}$)



wood ($c_1 \sim 10^{-4}$)



Nonlinear damping (from metal to wood):

$C(x) = c_0(x)I + c_1(x)K$ with
$c_n(x) = \beta_n\big(H(x)\big) \in [c_n^-, c_n^+]$

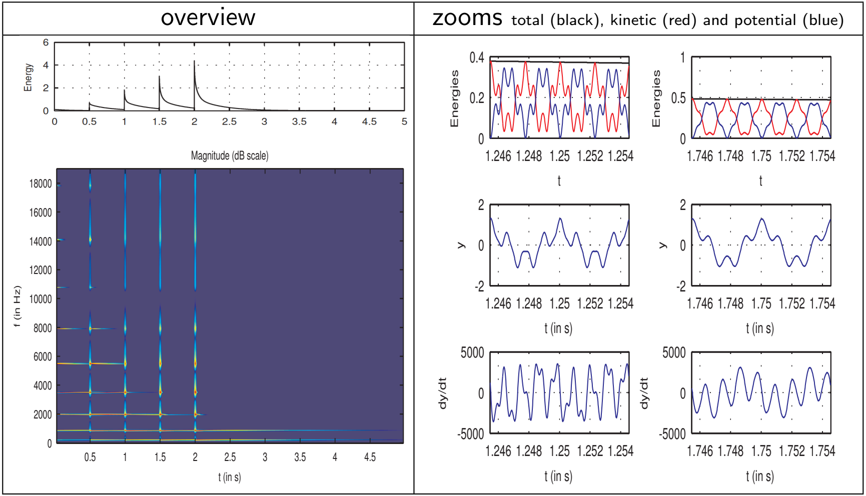
metal	$c_0^- = 0.02$	$c_1^- = 10^{-6}$
wood	$c_0^+ = 0.04$	$c_1^+ = 10^{-4}$

Numerical method preserving the power balance (discrete gradient)

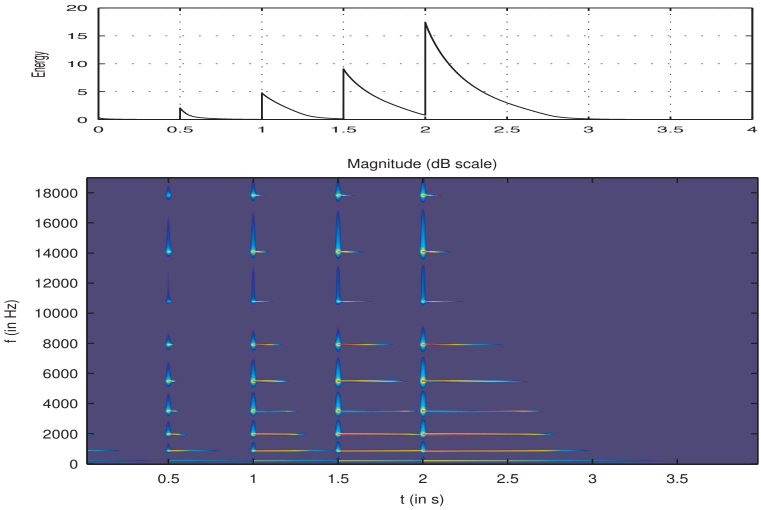
- force distributed close to $z = 0$: $u = [1, \dots, 1]^T f$
- listened signal: acceleration $[1, \dots, 1]\dot{y}$
- $n = 9$ modes and time step s.t. $f_1 = 220\,\text{Hz}$ to $f_9 \approx n^2 f_1 = 17820\,\text{Hz}$

(Results) Case 1: $E \ll 1 \longrightarrow$ metal, $E \gg 1 \longrightarrow$ wood

force: 5 piecewise constant pulses (0.1ms) with increasing magnitude



(Results) Case 2: $E \ll 1 \longrightarrow$ wood , $E \gg 1 \longrightarrow$ metal



Conclusion

For linear mechanical problems

Theoretical

- Port-Hamiltonian Formulation (power balance)
- class of **parametrized nonlinear damping** models
- preservation of the **original eigenstructure**

Practical *(sound synthesis of vibraphone, xylophone, marimba, etc)*

(a) **perceptively relevant variety** of sounds with a very few parameters

(b) **control of the dissipation properties** (only) according to the magnitude (here, the total energy),

(a,b) $\longrightarrow$ **morphed sounds** based on physical grounds

$\longrightarrow$ More technical details in [Hélie, Matignon, IFAC-LHMNLC'2015]

Conclusion

Some perspectives

Theoretical: Extensions to

- **non ideal** boundary conditions
- **non polynomial** operators $\mathcal{C}$ (rational functions, etc)
- some classes of **nonlinear conservative problems**

Applications: Examination of

- **1D/2D models of musical interest** (strings, plates, shells)
- **other nonlinear dampings** relevant for musical purposes

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