Port-Hamiltonian Systems for sound synthesis

Open avenues

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Workshop on Physical Sound Synthesis:
Ness/STMS/LAM-IJLRA

IRCAM, Paris, France, October 2016

Motivation

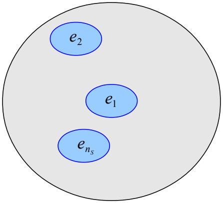
1. **Model input-output multi-physical systems** for sound and musical applications:
    - Phenomena: mechanical, acoustic, electronic, magnetic, etc
    - Realism: nonlinearities, non ideal dissipations, etc
2. **Satisfy fundamental physical properties**:
    - causality, stability, passivity and more precisely ...
    - the **power balance** structured into conservative/dissipative/source parts
    - other natural invariants and symmetries (if any)
3. **Simulate such systems and preserve these properties** in the discrete time domain *(+accuracy+anti-aliasing)*
4. **Design code generators** from netlists for real-time applications
5. **Design correctors and controllers** to reach target behaviors

Outline

Port Hamiltonian Systems:

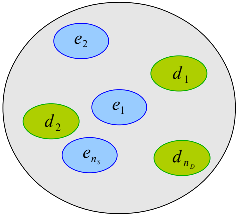
modelling input/output multi-physical passive systems

A physical system is made of ...



- Energy-storing components: *(energy)*
  $E = \sum_{n=1}^{N} e_n \geq 0$

A physical system is made of ...



- Energy-storing components: *(energy)*
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- Dissipative components: *(dissipated power)*
  $Q = \sum_{m=1}^{M} d_m \geq 0$

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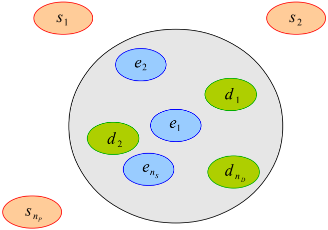

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  $E = \sum_{n=1}^{N} e_n \geq 0$
- Dissipative components: *(dissipated power)*
  $Q = \sum_{m=1}^{M} d_m \geq 0$
- External sources: *(external power)*
  $P_{\text{ext}} = \sum_{p=1}^{P} s_p$

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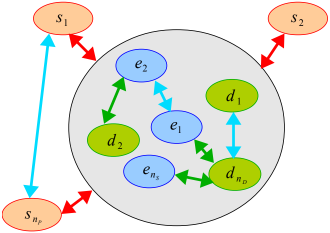

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- External sources: *(external power)*
  $P_{\mathrm{ext}} = \sum_{p=1}^{P} s_p$
- Conservative connections *(power balance)*
  $\frac{dE}{dt} = -Q + P_{\mathrm{ext}}$

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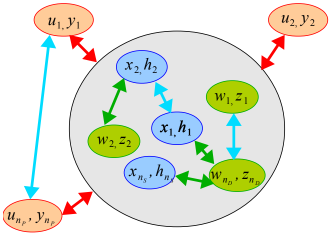
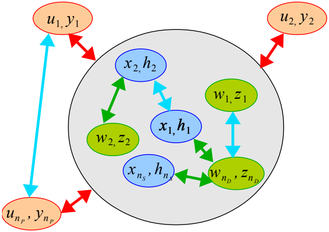


- Energy-storing components: *(energy)*
  $E = H(\mathbf{x}) = \sum_{n=1}^{N} H_n(x_n) \geq 0$
- Dissipative components: *(dissipated power)*
  $Q = \mathbf{z}(\mathbf{w})^{\mathbf{T}}\mathbf{w} = \sum_{m=1}^{M} z_m(w_m)\, w_m \geq 0$
  (effort × flux : force × velocity, voltage × current, etc)
- External sources: *(external power)*
  $P_{\mathrm{ext}} = \mathbf{u}^T \mathbf{y} = \sum_{p=1}^{P} u_p y_p$
- Conservative connections *(power balance)*
  $0 = \nabla H(x)^T \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} + \mathbf{z}(\mathbf{w})^{\mathbf{T}} \cdot \mathbf{w} - \mathbf{u}^T \cdot \mathbf{y}$

A physical system is made of ...



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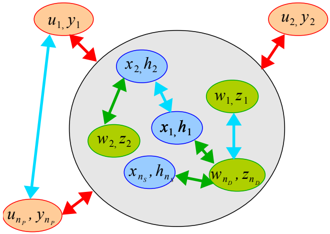
Port-Hamiltonian Formulation — **Power balance**

$$\begin{pmatrix} \frac{d\mathbf{x}}{dt} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix} = S. \begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}$$

A physical system is made of ...



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- **External sources:** *(external power)*
  $P_{\text{ext}} = \mathbf{u}^T\mathbf{y} = \sum_{p=1}^{P} u_p y_p$
- **Conservative connections** *(power balance)*
  $0 = \nabla H(x)^T \frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} + \mathbf{z}(\mathbf{w})^{\mathsf{T}}\cdot\mathbf{w} - \mathbf{u}^T\cdot\mathbf{y}$

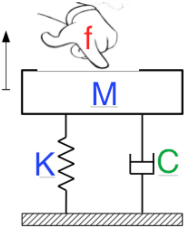
Port-Hamiltonian Formulation — **Power balance**

$$\underbrace{\begin{pmatrix} \frac{d\mathbf{x}}{dt} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix}}_{B} = S.\underbrace{\begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}}_{A}$$

$$\boxed{\begin{aligned} 0 &= A^T B \\ &= A^T S A \end{aligned}}$$

if $S = -S^T$

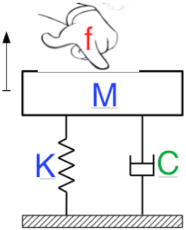
Example: damped mechanical oscillator



$x = \begin{bmatrix} M\dot{q} \\ q \end{bmatrix}$	*momentum* *position*	$H(x) = \frac{1}{2} x^T \begin{bmatrix} M^{-1} & 0 \\ 0 & K \end{bmatrix} x$	*kinetic* *potential*
$\frac{\mathrm{d}x}{\mathrm{d}t} = \begin{bmatrix} M\ddot{q} \\ \dot{q} \end{bmatrix}$	*inertia force* *velocity*	$\nabla H(x) = \begin{bmatrix} \dot{q} \\ Kq \end{bmatrix}$	*velocity* *spring force*
$w = \dot{q}$	*velocity*	$z(w) = C\dot{q}$	*damping force*
$y = \dot{q}$	*velocity*	$u = f$	*external force*

$$M\ddot{q} + C\dot{q} + Kq = f$$

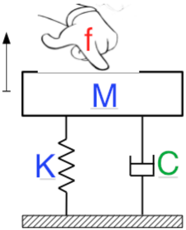
Example: damped mechanical oscillator



$x = \begin{bmatrix} M\dot{q} \\ q \end{bmatrix}$	momentum position	$H(x) = \frac{1}{2} x^T \begin{bmatrix} M^{-1} & 0 \\ 0 & K \end{bmatrix} x$	kinetic potential
$\frac{\mathrm{d}x}{\mathrm{d}t} = \begin{bmatrix} M\ddot{q} \\ \dot{q} \end{bmatrix}$	inertia force velocity	$\nabla H(x) = \begin{bmatrix} \dot{q} \\ Kq \end{bmatrix}$	velocity spring force
$w = \dot{q}$	velocity	$z(w) = C\dot{q}$	damping force
$y = \dot{q}$	velocity	$u = f$	external force

$$M\ddot{q} + C\dot{q} + Kq = f \longleftarrow \begin{pmatrix} \frac{\mathrm{d}x}{\mathrm{d}t} \\ \hline w \\ \hline -y \end{pmatrix} = \underbrace{\left(\begin{array}{cc|c|c} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \end{array}\right)}_{S=-S^T} \cdot \begin{pmatrix} \nabla H(x) \\ \hline z(w) \\ \hline u \end{pmatrix}$$

Example: damped mechanical oscillator



$x = \begin{bmatrix} M\dot{q} \\ q \end{bmatrix}$ momentum, position	$H(x) = \frac{1}{2} x^T \begin{bmatrix} M^{-1} & 0 \\ 0 & K \end{bmatrix} x$ kinetic, potential
$\frac{dx}{dt} = \begin{bmatrix} M\ddot{q} \\ \dot{q} \end{bmatrix}$ inertia force, velocity	$\nabla H(x) = \begin{bmatrix} \dot{q} \\ Kq \end{bmatrix}$ velocity, spring force
$w = \dot{q}$ velocity	$z(w) = C\dot{q}$ damping force
$y = \dot{q}$ velocity	$u = f$ external force

$$M\ddot{q} + C\dot{q} + Kq = f \longleftarrow \begin{pmatrix} \frac{dx}{dt} \\ w \\ -y \end{pmatrix} = \underbrace{\begin{pmatrix} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ +1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}}_{S=-S^T} \cdot \begin{pmatrix} \nabla H(x) \\ \mathbf{z}(w) \\ u \end{pmatrix}$$

N coupled oscillators ? q: vector

Matrices: $M = M^T > 0$, $K = K^T \geq 0$, $C = C^T \geq 0$, $1 \equiv I_N$

Some variations

Hamiltonian systems (conservative, autonomous)

$$\begin{pmatrix} F_M \\ v_K \\ \hline \cdot \\ \hline \cdot \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & -1 & \cdot & \cdot \\ +1 & 0 & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \end{array}\right) \cdot \begin{pmatrix} v_M \\ F_K \\ \hline \cdot \\ \hline \cdot \end{pmatrix}$$

"Mass+Damper+Excitation"

$$\begin{pmatrix} F_M \\ \cdot \\ \hline v_C \\ \hline -v_{\text{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & \cdot & -1 & +1 \\ \hline \cdot & \cdot & \cdot & \cdot \\ \hline +1 & \cdot & 0 & 0 \\ \hline -1 & \cdot & 0 & 0 \end{array}\right) \cdot \begin{pmatrix} v_M \\ \cdot \\ \hline F_C \\ \hline F_{\text{ext}} \end{pmatrix}$$

"Mass+Excitation"

$$\begin{pmatrix} F_M \\ \cdot \\ \hline \cdot \\ \hline -v_{\text{ext}} \end{pmatrix} = \left(\begin{array}{cc|c|c} 0 & \cdot & \cdot & +1 \\ \hline \cdot & \cdot & \cdot & \cdot \\ \hline \cdot & \cdot & \cdot & \cdot \\ \hline -1 & \cdot & \cdot & 0 \end{array}\right) \cdot \begin{pmatrix} v_M \\ \cdot \\ \hline \cdot \\ \hline F_{\text{ext}} \end{pmatrix}$$

Differential formulation

Formulation 1: differential-algebraic

$$\begin{pmatrix} \frac{dx}{dt} \\ \hline \mathbf{w} \\ \hline -\mathbf{y} \end{pmatrix} = S \cdot \begin{pmatrix} \nabla H(\mathbf{x}) \\ \hline \mathbf{z}(\mathbf{w}) \\ \hline \mathbf{u} \end{pmatrix}, \qquad S = -S^T$$

Formulation 1→ 2: solving algebraic part $w = W(\nabla H(x), u)$

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = (J-R)\nabla H(x) + Gu, & J=-J^T, \quad R = R^T \geq 0 \\ -y = -G^T \nabla H(x) + Du, & D = -D^T \end{cases}$$

"Mass-Spring-Damper": $H(x) = \frac{x_1^2}{2M} + \frac{K x_2^2}{2}$, $z(w) = Cw$

$$S = \left[\begin{array}{cc|c|c} 0 & -1 & -1 & +1 \\ +1 & 0 & 0 & 0 \\ \hline +1 & 0 & 0 & 0 \\ \hline -1 & 0 & 0 & 0 \end{array}\right], \quad J = \begin{bmatrix} 0 & -1 \\ +1 & 0 \end{bmatrix}, \ R = \begin{bmatrix} C & 0 \\ 0 & 0 \end{bmatrix}, \ G = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ D = 0$$

Guaranteed-Passive Simulation:

How to preserve the power-balance ?

Numerical method: basic principle

Approach *(formulation 2)*

- **Classical case** $\frac{\mathrm{d}x}{\mathrm{d}t} = f(x)$**:** efficiently approximate $\frac{\mathrm{d}.}{\mathrm{d}t}$ and exploit f
- **PHS case:** preserve the power balance in the discrete time-domain

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \nabla H(x)^T \frac{\mathrm{d}x}{\mathrm{d}t} = \underbrace{\nabla H(x)^T J(x) \nabla H(x)}_{P_c=0} - \underbrace{\nabla H(x)^T R(x) \nabla H(x)}_{Q\geq 0} + \underbrace{y^T u}_{P_{\mathrm{ext}}}$$

How? (1) differentiation chain rule for $E = H \circ x$; (2) exploit J and R

(1) Choice: $\frac{E[k+1]-E[k]}{\delta T} = \sum_{n=1}^{N} \frac{H_n(x_n[k+1])-H_n(x_n[k])}{x_n[k+1]-x_n[k]} \cdot \frac{x_n[k+1]-x_n[k]}{\delta T}$

(2) Substitutions: $\frac{\mathrm{d}x}{\mathrm{d}t} \to \frac{\delta x[k]}{\delta T} = \frac{\mathbf{x}[k+1]-\mathbf{x}[k]}{\delta T}$ and $\nabla H(x) \to \nabla_d H\big(x[k], \delta x[k]\big)$

with $[\nabla_d H(x, \delta x)]_n = \dfrac{H_n(x_n + \delta x_n) - H_n(x_n)}{\delta x_n}$ if $\delta x_n \neq 0$ and $H_n'(x_n)$ otherwise

This method splits the approximation in two parts:

1. differential operator; 2. vector field $f = (J - R)\nabla H \to f_d = (J - R)\nabla_d H$.

Numerical method

Formulation 1: $x[k+1] = x[k] + \delta x[k]$ and solve:

$$\begin{cases} \dfrac{\delta x[k]}{\delta t} &= (J-R)\ \nabla_d H(x[k], \delta x[k]) \ + G u[k] \\ y[k] &= G^T\ \nabla_d H(x[k], \delta x[k]) \end{cases}$$

Case of linear systems $H(x) = \frac{1}{2} x^T W x$ avec $W = W^T > 0$

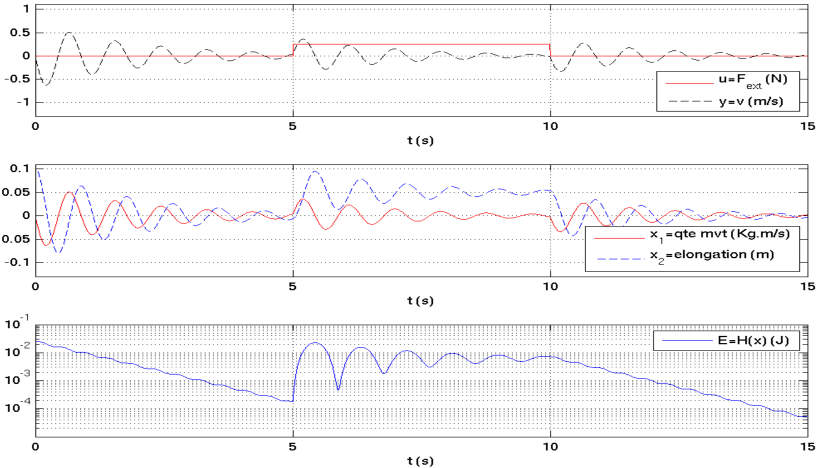
Mid-point: $\nabla_d H(x[k], \delta x[k]) = W\left(x[k] + \frac{\delta x[k]}{2}\right) = \nabla H\left(\frac{x[k]+x[k+1]}{2}\right)$

We can show that: [Aoues, Thèse, 2014] & [Lopes et al., IFAC-LHMNLC'2015]

- (A) Discrete gradient: still available for multi-variate functions H,
- (L) Consistancy: in general, order 1; 2 if J and R do not depend on (x, w),
- (L) Order 2 (or higher) can be reached (Runge-Kutta-like refinements),
- (L) Existence, uniqueness, non-iterative solver: if H is convex.

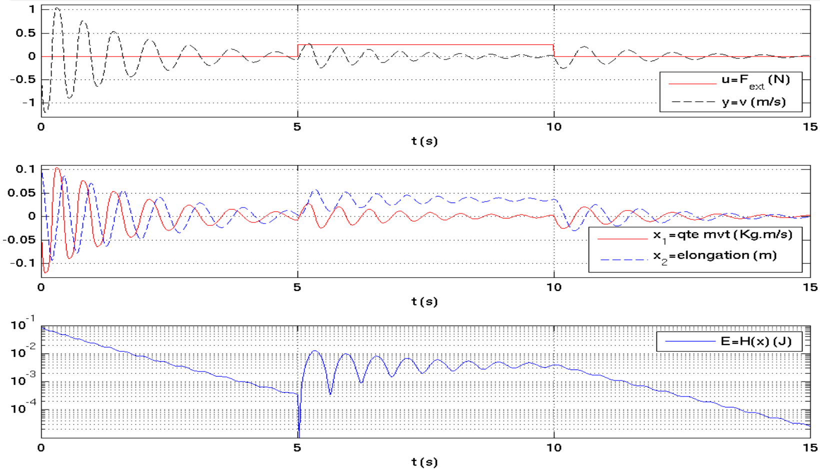
Simulation 1: mass-spring-damper

- **Parameters:** $M = 100\,\mathrm{g}$, $K = 3\,\mathrm{kN/m}$, $C = 0.1\,\mathrm{N.s/m}$ et $\delta t = 5\,\mathrm{ms}$
- **Initial conditions:** $x_0 = \left[m v_0 = 0,\ \ell_0 = 10\,\mathrm{cm} \right]^T$
- **Excitation:** $F_{\mathrm{ext}}(t) = F_{\mathrm{max}}\, \mathbf{1}_{[5\mathrm{s},10\mathrm{s}]}(t)$ with $F_{\mathrm{max}} = K\ell_0/2 = 0.25\,\mathrm{N}$



Simulation 2: idem with a hardening spring

- **Potential energy:** $H_2^{\mathrm{NL}}(x_2) = K\,L^2\left[\cosh(x_2/L) - 1\right] \quad \left(\sim k\,x_2^2/2\right)$
- **Physical law:** $F_2 = \left(H_2^{\mathrm{NL}}\right)'(x_2) = K\,L\,\sinh(x_2/L) \quad \left(\sim K\,x_2\right)$
- **Reference elongation:** $L = \ell_0/4 = 25\,\mathrm{mm}$



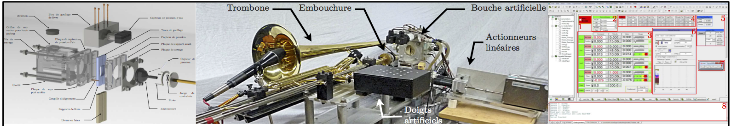
Recent results:

- **Mutating materials** in beams for physical sound morphing
  $\rightarrow$ Extension of Caughey series to **damping classes** for PDEs
  (Hélie, Matignon: ViennaTalk'2015)
  Sound 1 ($E << E_0$: wood , $E >> E_0$: metal)
- **Damped nonlinear string and exact order reduction**
  (Hélie, Roze: CFA'2016)
  Sound 2 (large distributed sawtooth force)
- **Brass instruments** (PhD'2016)
- **Audio systems** (PhD'2016)
- **A simplified but complete vocal apparatus**
  (Hélie, Silva: Summer School on Voice'2016)

PhD, June 2016: Nicolas Lopes

Passive modelling, simulation and experimental study of a robotised artificial mouth playing brass instruments

1. PHS/Simu of the complete system: *air jet in a channel with mobile walls, etc*



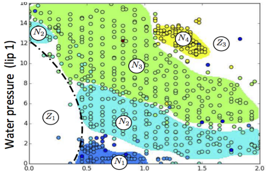
Instrumented mouth, lips and mouthpiece

Robotised testbed

HMI *(dSpace & Python)*

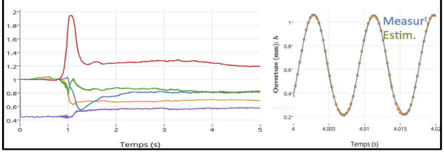
2. Automated exploration

Analysis of notes (N_j)



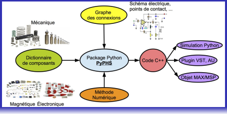
3. Estimation/Observation: extended Kalman filter on PHS

(5 estimated parameters: lip mass, stifness, *etc*)



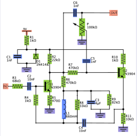
PhD, July 2016: Antoine Falaize

Passive modelling, simulation, code generation
and correction of audio multi-physical systems



Two examples (theory+other applications in the manuscript)

Wah pedal (CryBaby): netlist → PyPHS → LateX eq. & Simulation C code



Components	Number
Storage	7 linear
Dissipative	18 (5 NL, 2 modulated)
Ports	3 (IN, OUT, battery)

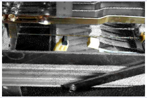
Audio PlugIn:

Sound 3a: dry

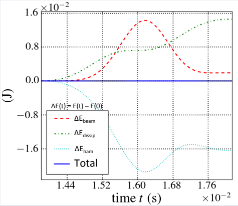
Sound 3b: wah

A simplified Fender-Rhodes Piano

Sounds 4a-c



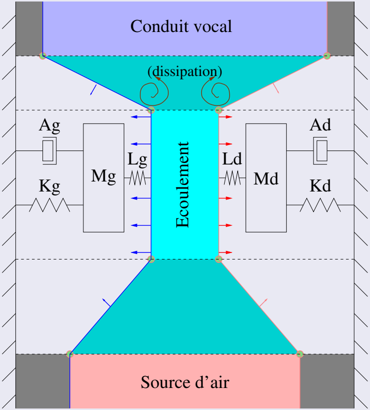
Components	Hammer	1 beam	Pickup/RC-circuit
Storage	2 NL	$2M$ lin.	2 lin. (+ NL connection)
Dissipative	1 NL	M lin.	1 lin.
Ports	2	1	1



Loudspeaker and correction (+see next presentation)

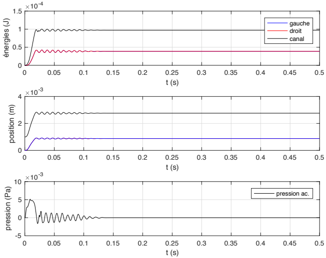
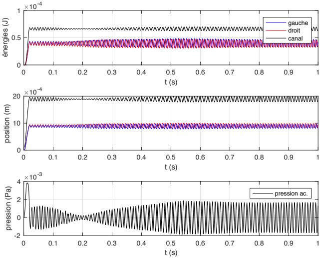
A simplified but complete vocal apparatus

(Hélie, Silva'2016: Summer School on Voice, two weeks ago)

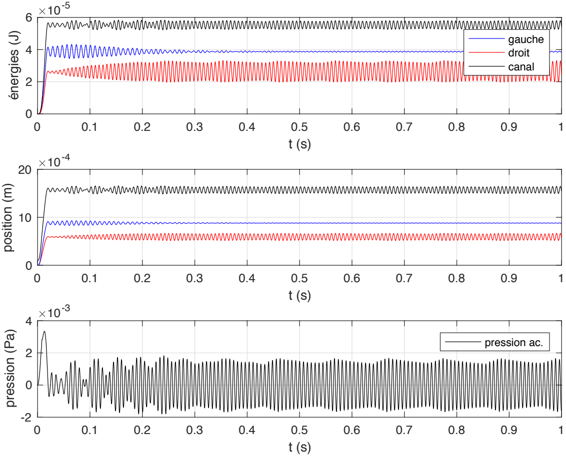


- Acoustic resonator: 1 formant
- Turbulences: dissipation of the kinetic energy of the jet
- Vocal folds (x2):
  mass-spring-damper
  + spring connected to the channel
- Air flow: lossless perfect gas, incompressible, irrotational, in a straight channel with mobile walls
- upstream/downstream: pressure forces on the conical walls
- Source: controlled pressure supply

Phonation: simulation (standard parameters)

$f_l = f_r = 112\text{Hz},\ h_0 = 1\text{mm}$	110Hz, 112Hz, 0.1mm
	

Phonation: simulation (large dissymmetry: $f_l = 112\text{Hz}$, $f_r = 137\text{Hz}$)



Perspectives

A few perspectives

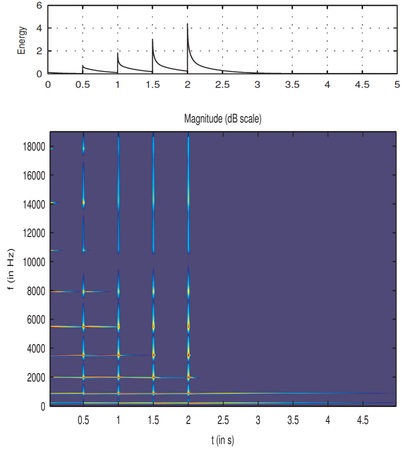
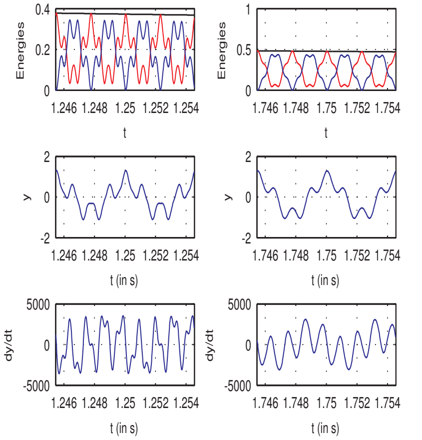
- **Modelling**: differential geometry/invariance/symmetries *(last presentation)*, thermodynamics, energy-balanced Lattice Boltzman Networks ($\neq$ CORDIS)
- **Numerical methods**
  - Consistency analysis (/dispersion study ?)
  - Anti-aliasing for nonlinearities
  - Links with Wave Digital Filters

  $\longrightarrow$ Remy Muller's PhD
- **PyPHS**
  - Dictionary of components: to be completed...
  - Causality conflicts (capa//capa) $\rightarrow$ model order reduction
  - FAUST code generation and automatic parallelisation (open problem)
- **Observation, correction and control issues** *(next presentation)*
- **Realistic vocal apparatus**:
  - Modelling
  - Simulation
  - Control
  - Robotised testbed $\rightarrow$

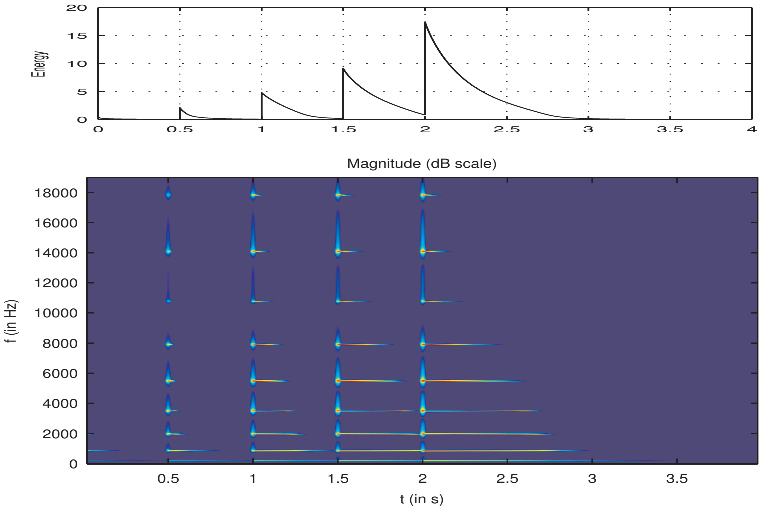
APPENDIX

(Beam) Case 1: $E \ll 1 \longrightarrow$ metal, $E \gg 1 \longrightarrow$ wood

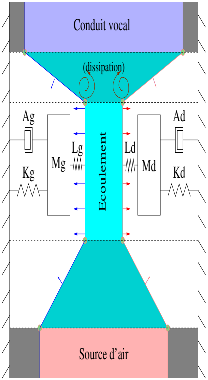
force: 5 piecewise constant pulses (0.1ms) with increasing magnitude

overview	zooms total (black), kinetic (red) and potential (blue)
	

(Beam) Case 2: $E \ll 1 \longrightarrow$ wood , $E \gg 1 \longrightarrow$ metal



Le système complet



$$
\begin{bmatrix} \dot{x}_d \\ \dot{\pi}_d \\ \dot{\eta}_d \\ \hline \dot{x}_g \\ \dot{\pi}_g \\ \dot{\eta}_g \\ \hline \dot{\pi}_1 \\ \dot{\pi}_2 \\ \dot{\pi}_3 \\ \dot{h} \\ \hline \dot{X}_{ac1} \\ \dot{X}_{ac2} \\ \hline w_{turb} \\ \hline -Q_{sub} \end{bmatrix}
=
\left[\begin{array}{ccc|ccc|cccc|cc|c|c}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & -a_d & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & S^d_{sup}1_N^T & 0 & 0 & S^d_{sub} \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & \frac{h_0}{h} & -1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -a_g & -1 & 0 & 0 & 0 & 0 & S^g_{sup}1_N^T & 0 & 0 & S^g_{sub} \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & -\frac{h_0}{h} & -1 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -L_0 h_0 1_N^T & 0 & -L_0 h_0 & L_0 h_0 \\
0 & 0 & -\frac{h_0}{h} & 0 & 0 & \frac{h_0}{h} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & -2 & 2L_0\ell_0 1_N^T & 0 & 2L_0\ell_0 & 2L_0\ell_0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & -S^d_{sup}1_N & 0 & 0 & -S^g_{sup}1_N & 0 & L_0 h_0 1_N & 0 & -2L_0\ell_0 1_N & 0 & -R_{ac} & -\mathbb{I}_N & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbb{I}_N & 0 & 0 & 0 \\
\hline
0 & 0 & 0 & 0 & 0 & 0 & L_0 h_0 & 0 & -2L_0\ell_0 & 0 & 0 & 0 & 0 & 0 \\
\hline
0 & -S^d_{sub} & 0 & 0 & -S^g_{sub} & 0 & -L_0 h_0 & 0 & -2L_0\ell_0 & 0 & 0 & 0 & 0 & 0
\end{array}\right]
\begin{bmatrix} \partial_{x_d} H \\ \partial_{\pi_d} H \\ \partial_{\eta_d} H \\ \hline \partial_{x_g} H \\ \partial_{\pi_g} H \\ \partial_{\eta_g} H \\ \hline \partial_{\pi_1} H \\ \partial_{\pi_2} H \\ \partial_{\pi_3} H \\ \partial_h H \\ \hline \nabla_{X_{ac1}} H \\ \nabla_{X_{ac2}} H \\ \hline z_{turb} \\ \hline P_{sub} \end{bmatrix}
$$

Où se cachent les non-linéarités ?

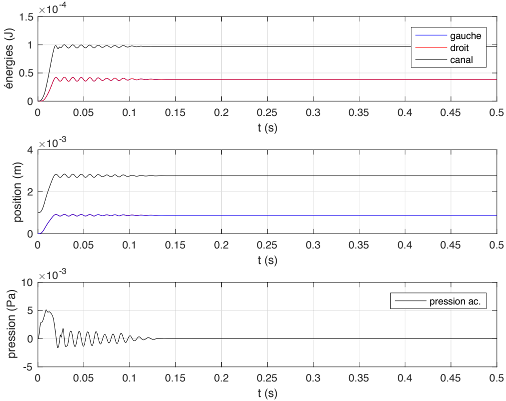
1. dans l'énergie de l'écoulement ($\to$ auto-oscillation);
2. dans les pertes par turbulences (z)

Simulations, tests et interprétations

Paramètres

- Plis : masse $0.2g$, amortissement $r = 0.05$, raideur de couche superficielle $L = 3K$
- Canal glottique de largueur $L_0 = 11mm$, longueur $4mm$ hauteur au repos $0.1mm$
- Surfaces exposées aux pressions supraglottique $1.1mm^2$ et subglottique $1.1cm^2$
- Formant /a/ russe [Badin, 1984] : 640Hz, facteur de qualité 2.5, amplitude $1M\Omega$
- Pression subglottique $800Pa$ avec une montée de $20ms$

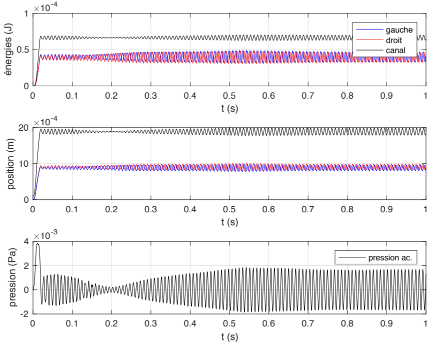
Configuration 1 : Faible adduction



Hauteur au repos $1mm$
Plis symétriques
raideur $100N/m$
Fréquence de résonances $112Hz$

Pas d'oscillation

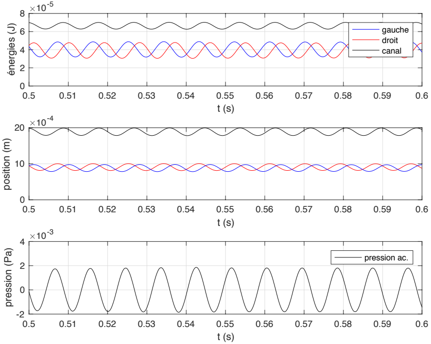
Configuration 2 : Meilleure adduction



Hauteur au repos 0.1mm
Plis quasi symétriques
raideurs 100N/m et 97N/m
Fréquence de résonances 112Hz et 110Hz

Oscillation stabilisée après un transitoire de l'ordre de 0.4ms

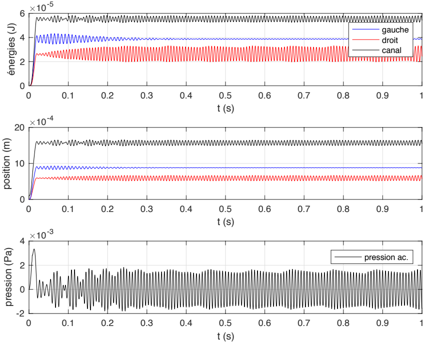
Configuration 2 : Meilleure adduction



Hauteur au repos $0.1mm$
Plis quasi symétriques
raideurs $100N/m$ et $97N/m$
Fréquence de résonances $112Hz$ et $110Hz$

Les plis sont bien synchronisés en régime permanent (sans contact).

Configuration 3 : forte asymétrie



Hauteur au repos $0.1mm$
Plis asymétriques
raideurs $100N/m$ et $149N/m$
Fréquence de résonances $112Hz$ et $137Hz$
Une oscillation s'établit avec modulation, portée au début par le pli le plus mou durant le transitoire, puis par le pli le plus raide (et médialisé).