Simulation and correction of an electrodynamic loudspeaker based on Port-Hamiltonian Systems

(internship results)

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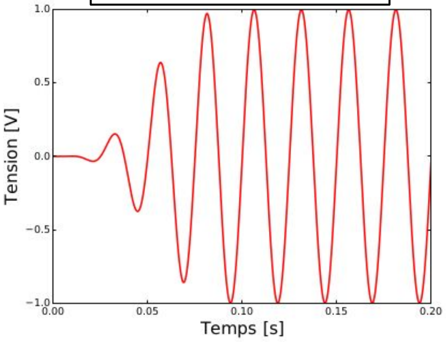
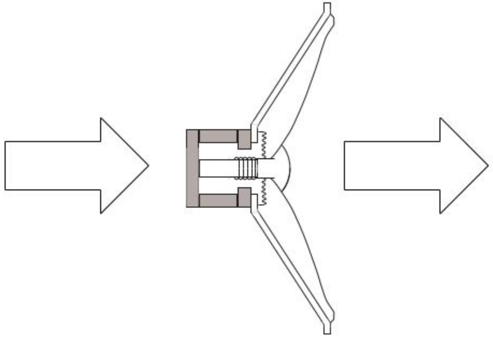
Motivation (1/2)

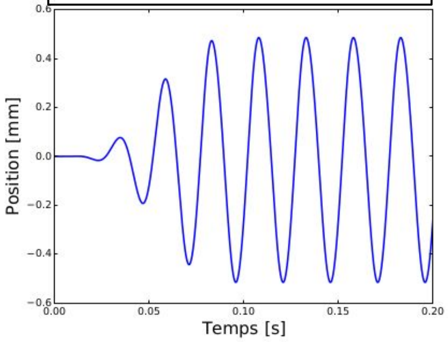
Low-level excitation

VOLTAGE VS TIME



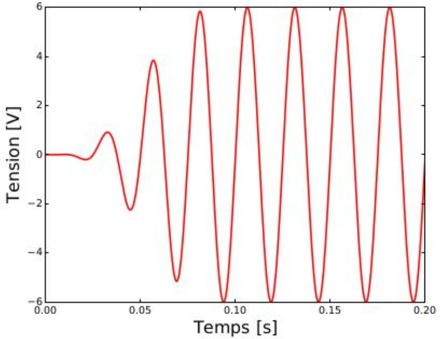


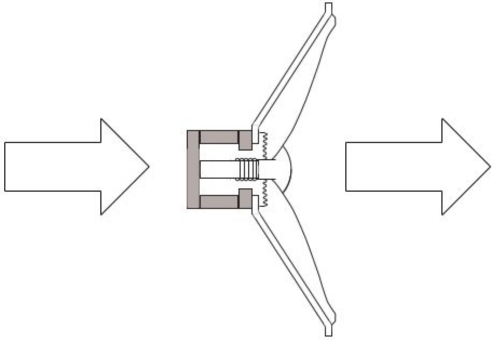
DISPLACEMENT VS TIME



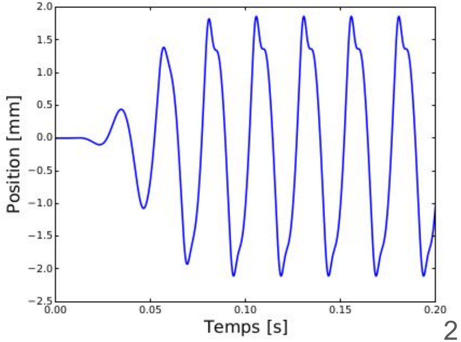
High-level excitation : distorsions

VOLTAGE VS TIME

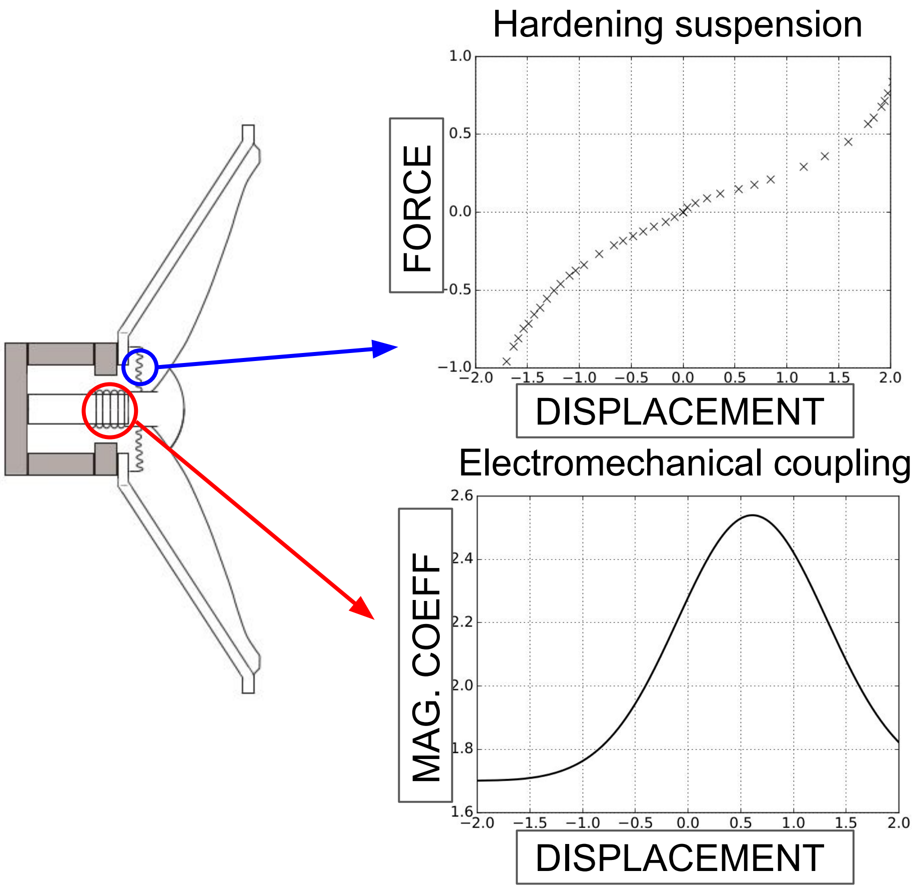




DISPLACEMENT VS TIME



Motivation (2/2)



Goal :
Compensation of the nonlinearities

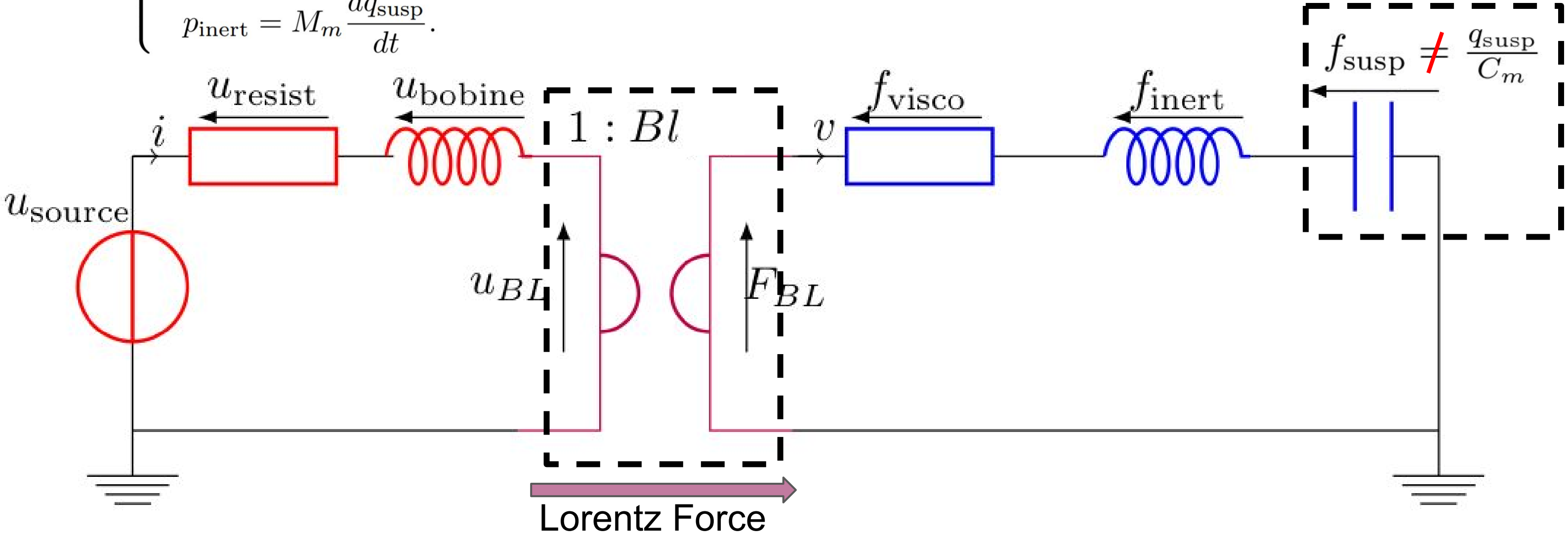
Summary

I. Modeling and Port-Hamiltonian formalism
II. Passive simulation (sounds)
III. Flatness-based correction
IV. Experimentations on a test bench

I. Modeling and Port-Hamiltonian formalism

Model : Thiele & Small **+ nonlinear refinements**

$$\begin{cases} u_{\text{source}} = R_{\text{e}} \dfrac{\phi_{\text{bobine}}}{L_{\text{e}}} + \dfrac{d\phi_{\text{bobine}}}{dt} + Bl(q_{\text{susp}}) \dfrac{dq_{\text{susp}}}{dt}, \\ \dfrac{dp_{\text{inert}}}{dt} = \dfrac{Bl(q_{\text{susp}})}{L_{\text{e}}} \phi_{\text{bobine}} - R_{\text{m}} \dfrac{dq_{\text{susp}}}{dt} - f_{\text{susp}}(q_{\text{susp}}), \\ p_{\text{inert}} = M_m \dfrac{dq_{\text{susp}}}{dt}. \end{cases}$$



Formalism : Port-Hamiltonian Systems

- Energy of a system : $\mathcal{H}(X) \geq 0$ with $X(t)$ state
- Dissipated power : $\mathcal{D} = \underbrace{Z(\mathrm{w})^T}_{R.i} \underbrace{\mathrm{w}}_{i}$
- External power : $\mathcal{P}^{EXT} = \underbrace{\mathrm{u}^T}_{V} \underbrace{\mathrm{y}}_{i}$

→ Power exchanges encoded in PHS structure :

$$\begin{array}{l} \text{Conservative} \\ \text{Dissipative} \\ \text{Ports} \end{array} \underbrace{\begin{pmatrix} \frac{dX}{dt} \\ \mathrm{w} \\ -y \end{pmatrix}}_{A} = S \underbrace{\begin{pmatrix} \nabla\mathcal{H}(X) \\ Z(\mathrm{w}) \\ \mathrm{u} \end{pmatrix}}_{B}$$

S **skew-symmetric** : S = -S^T

Power balance :

$$\frac{dE}{dt} = -\mathcal{D} + \mathcal{P}_{EXT}$$

⇔ $B^TA = 0$

⇔ $B^TSB = 0$ ✔

I. Modeling and Port-Hamiltonian formalism

Port-Hamiltonian formalism : application to the loudspeaker

State : $X = \begin{pmatrix} q_{\text{susp}} \\ p_{\text{inert}} \\ \phi_{\text{bobine}} \end{pmatrix}$

- q_{susp} → Diaphragm displacement
- p_{inert} → Mass momentum
- ϕ_{bobine} → Magnetic flux induced in the coil

Energy : $\mathcal{H}(X) = h_{\text{susp}}(q_{\text{susp}}) + h_{\text{inert}}(p_{\text{inert}}) + h_{\text{bobine}}(\phi_{\text{bobine}})$

Dissipative laws :

$$Z(\mathrm{w}) = \begin{pmatrix} R_e.i_{\text{resist}} \\ R_m.v_{\text{visco}} \end{pmatrix}$$

External ports :

$\mathrm{u} = u_{\text{source}} \quad y = i_{\text{source}}$

$$\begin{pmatrix} \frac{dX}{dt} \\ \mathrm{w} \\ -y \end{pmatrix} \equiv \underbrace{\begin{pmatrix} v_{\text{susp}} \\ f_{\text{inert}} \\ u_{\text{bobine}} \\ \hline i_{\text{resist}} \\ v_{\text{visco}} \\ \hline -i_{\text{source}} \end{pmatrix}}_{A} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & Bl(q_{\text{susp}}) & 0 & -1 & 0 \\ 0 & -Bl(q_{\text{susp}}) & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix}}_{S} \underbrace{\begin{pmatrix} f_{\text{susp}} \\ v_{\text{inert}} \\ i_{\text{bobine}} \\ \hline u_{\text{resist}} \\ f_{\text{visco}} \\ \hline u_{\text{source}} \end{pmatrix}}_{B} \equiv \begin{pmatrix} \nabla\mathcal{H}(X) \\ Z(\mathrm{w}) \\ \mathrm{u} \end{pmatrix}$$

II. Numerical method for passive simulation

Aim : ensure stable simulations of passive systems

Method : preserve the power balance in the discrete-time domain

$$\delta_t E_k = -\mathcal{D}_k + \mathcal{P}_k^{EXT}$$

Derivative of the chain rule $E = \mathcal{H} \circ X$: $\dfrac{dE}{dt} = \dfrac{dX}{dt}.\nabla\mathcal{H}(X)$

1) Forward Euler numerical scheme : $\delta_t X_k = \dfrac{X_{k+1} - X_k}{T_e}$

2) Discrete version of the gradient operator :

$$[\nabla^d \mathcal{H}]_j(X_k, \delta X_k) = \begin{cases} \dfrac{h_j([X_k + \delta X_k]_j) - h_j([X_k]_j)}{[\delta X_k]_j} & if \quad [\delta X_k]_j \neq 0 \\ h'_j([X_k]_j) & if \quad [\delta X_k]_j = 0 \end{cases}$$

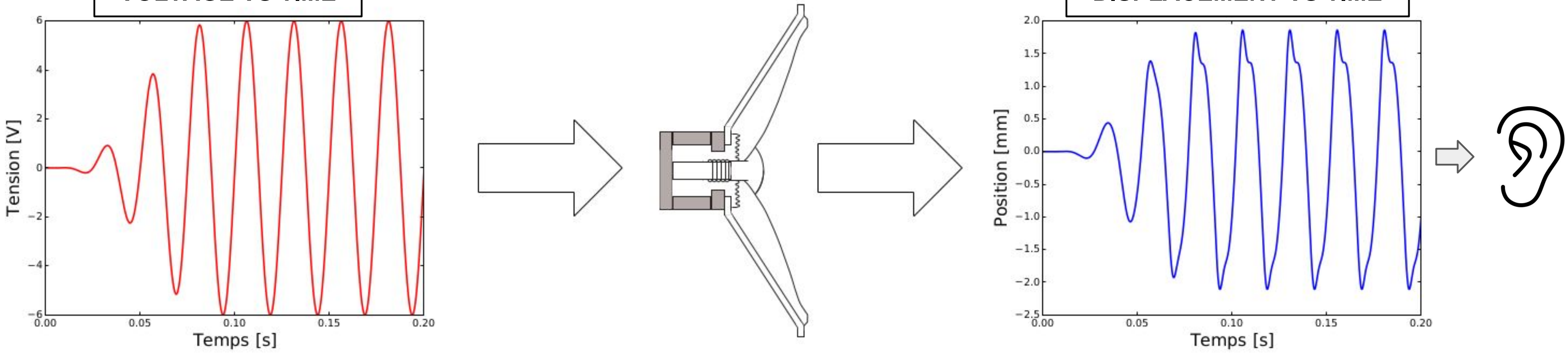
➡ Chain rule preserved in discrete time : $\delta_t E_k = \nabla^d \mathcal{H}(X_k, \delta_t X_k)^T . \delta_t X_k$

II. Numerical method for passive simulation

Results

- Solver based on Newton-Raphson iterative method

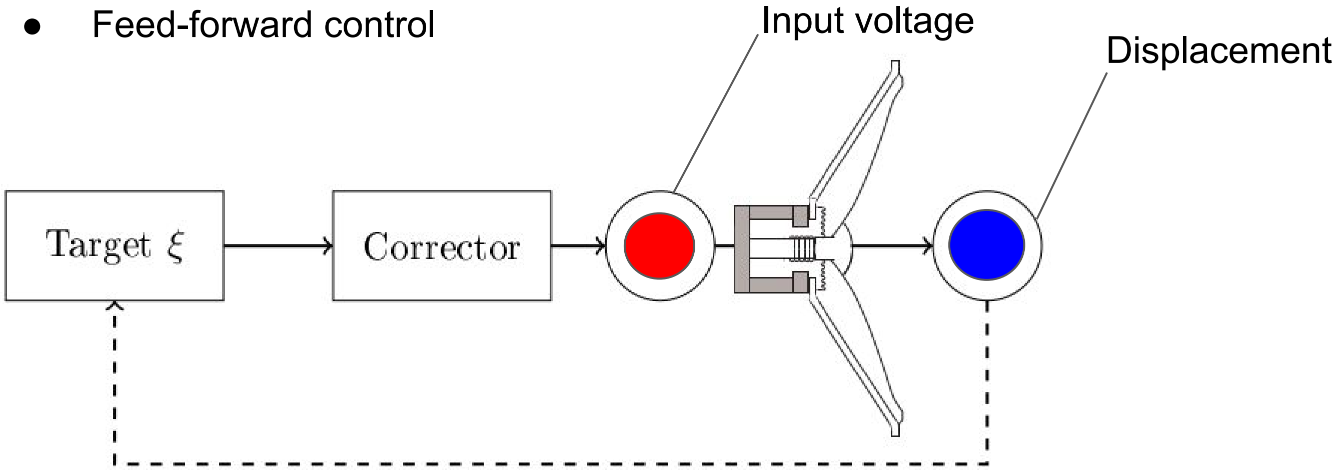
VOLTAGE VS TIME **DISPLACEMENT VS TIME**

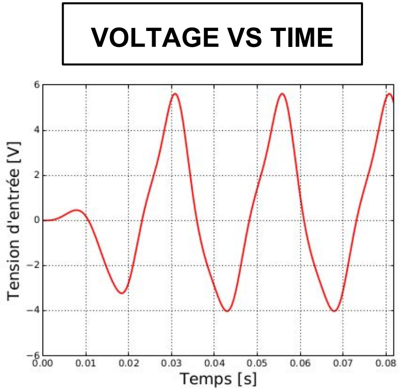


Listening 1 : Sweep signal, from 440 Hz to 20 Hz
Listening 2 : Bass guitar riff

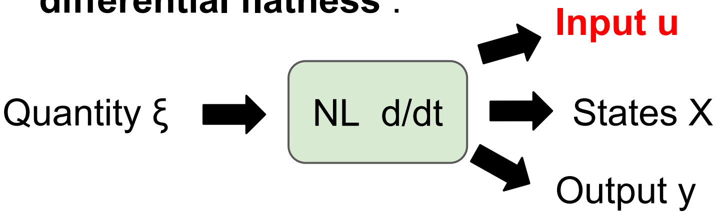
III. Flatness-based correction

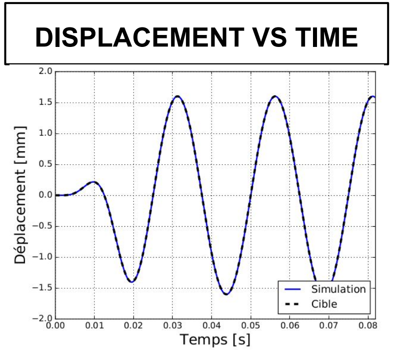
- Feed-forward control





- Corrector designed using **differential flatness** :



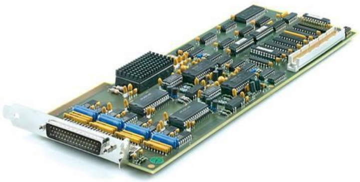


IV. Experimentations on a test-bench

Sensors:

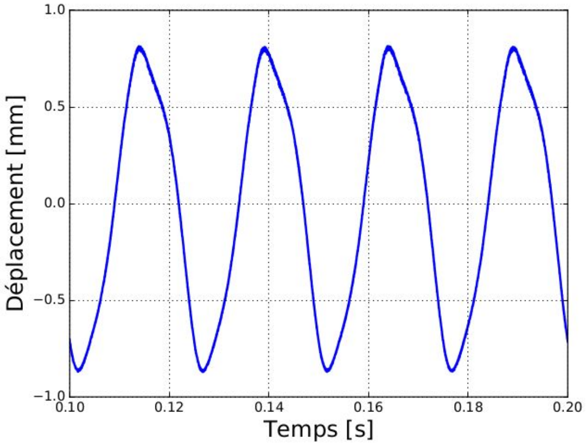
- Displacement
- Current
- Voltage

I/O dSPACE platform



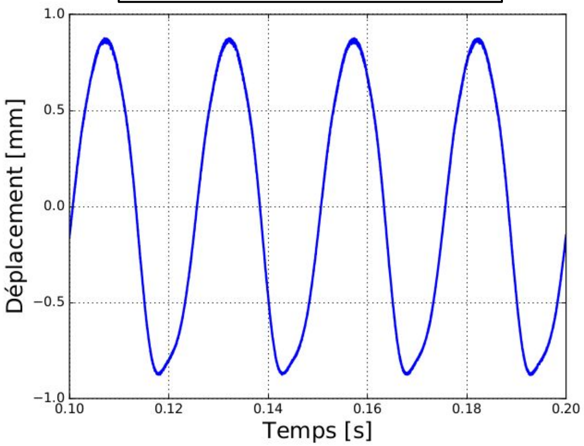
WITHOUT CORRECTION

DISPLACEMENT VS TIME



WITH CORRECTION

DISPLACEMENT VS TIME

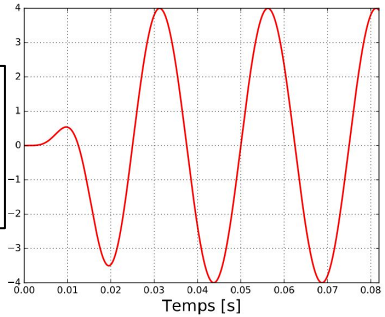


CONCLUSION

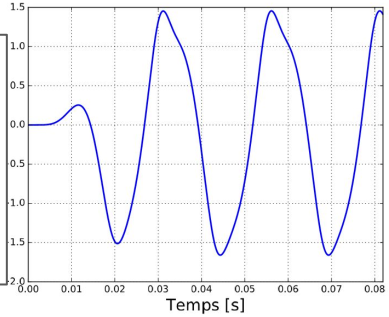
NUMERICAL RESULTS

WITHOUT CORRECTION

VOLTAGE

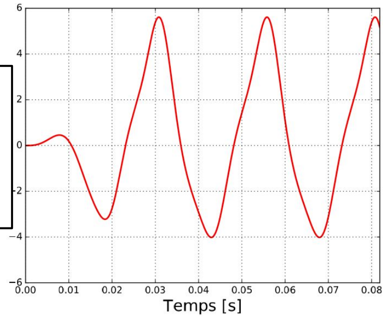


DISPLACEMENT

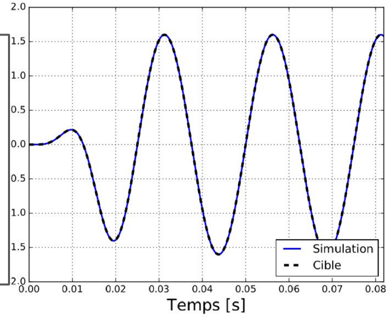


WITH CORRECTION

VOLTAGE



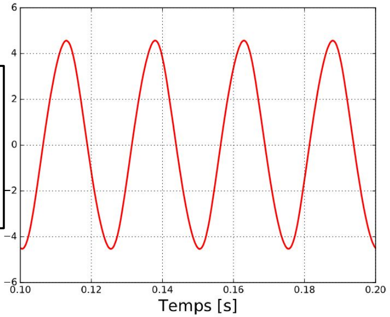
DISPLACEMENT



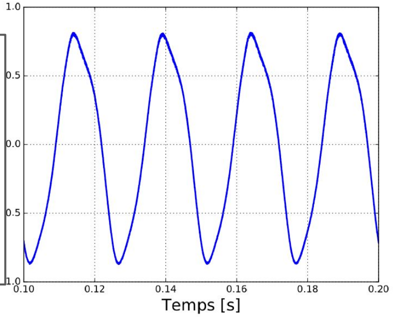
EXPERIMENTAL RESULTS

WITHOUT CORRECTION

VOLTAGE

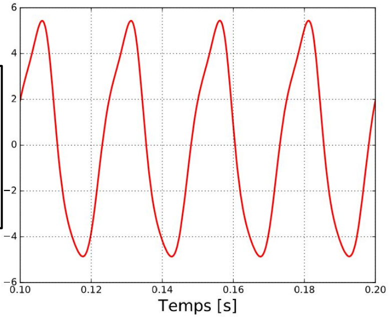


DISPLACEMENT



WITH CORRECTION

VOLTAGE



DISPLACEMENT